Statistical interpretation of seismic waves to elucidate the heterogeneity of the medium: Correlation functions and probability distributions

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Introduction

The seismic waves are distorted due to the scattering by the random heterogeneities in the medium. The distortions in the waveforms observed at different locations depend on the nature of heterogeneities lying across the wave paths between sources and receivers. Thus, the study of waveform distortions provides the information on the heterogeneities of the medium. We measured the elastic waveforms in different heterogeneous media and for different wave frequencies using a Laser Doppler Vibrometer (Nishizawa et al., 1997). About 180 waveforms have been measured over a circular array of radius 10 mm and a station spacing of 0.35 mm. The total distortion in the waveform can be represented by individual distortions such as arrival times, amplitudes and shape of the waveform. A statistical analysis of arrival times, amplitudes and shape of the waveforms is performed. The relationships between various statistical quantities obtained from the analysis of the waveform and the deterministic properties of the medium are inferred.

Autocorrelation function of the random media

The spatial distribution of seismic velocities or densities of the heterogeneities in a medium can be viewed as the fluctuation over a back ground value, which is an ensemble average of the physical property. A random medium characterized by the distribution of such fluctuations can be described by the spatial autocorrelation functions (ACF) such as:

$$F(x) = \varepsilon^2 \exp\left(\frac{-x}{a}\right) \qquad (1)$$

where ε^2 is the strength of the heterogeneity(velocity fluctuations) and a is the characteristic length of heterogeneity. We estimated the autocorrelation function from the velocity fluctuation data deduced from the microstructure images of three granite samples. The estimated AFC is then fitted by equation (1) to obtain the parameters ε and a. The estimated characteristic lengths of heterogeneity are 0.22, 0.46 and 0.92 mm for Westerly, Oshima and Inada granites respectively.

Coherency measures

The observed wave, $wo_i(t)$ can be written as summation of coherent wave, $wc_i(t)$, and the incoherent or random waves, $wr_i(t)$, generated from scattering by random heterogeneities in a medium, measured at station, i, at a given instant of time, t, as:

$$wo_i(t) = wc_i(t) + wr_i(t) \qquad (2)$$

The coherent wave is the ensemble average of all the waves in an array and can be viewed as undistorted signal. The correlation coefficient between coherent and observed wave (distorted wave) at *i-th* station in the array is then computed by

$$r_i = \frac{\sum_{t=T_s}^{Te}\left\{wo_i(t) - \overline{wo_i}\right\}\left\{wc_i(t) - \overline{wc_i}\right\}}{\sqrt{\sum_{t=T_s}^{T_e}\left\{wo_i(t) - \overline{wo_i}\right\}^2}\sqrt{\sum_{t=T_s}^{T_e}\left\{wc_i(t) - \overline{wc_i}\right\}^2}} \qquad (3)$$

where, $\overline{wo_i}$ and $\overline{wc_i}$ represent the average observed and coherent wave fields between starting time, T_s and the ending time, T_e. The average and variance of the correlation coefficients, r_i, are then computed for each array. We have also studied the spatial coherency between the waveforms measured at two different locations separated by a distance lag to understand their resemblance.

Estimation of energy decay pattern

We computed the wave energy in an appropriate time window by

$$E_i(t_m) = \sum_{t=t_1}^{t_2} A_i^2(t) \qquad (4)$$

where, $A_i(t)$ is the amplitude of the i^{th} waveform at time, t and t_m is the mean of starting (t_1) and ending (t_2) times of the window. The energy estimated in each window is normalized by the total energy of the waveform starting from P-wave onset to the end of the waveform. The total normalized energy of all waves in the array for each time window is computed by

$$En(t_m) = \sum_{i=1}^{N}\sum_{t=t_1}^{t_2} A_i^2(t) \Big/ \sum_{i=1}^{N}\sum_{t=t_p}^{T} A_i^2(t) \qquad (5)$$

where t_p is the arrival time of the P-wave, T is the total time of the waveform and N is the total number of waveforms in the array. The normalized energy $En(t_m)$ is computed by shifting the window, without any overlap, till the end of the waveform for observed and scattered waves and the decay pattern is studied as a function of lapse time, t_m.

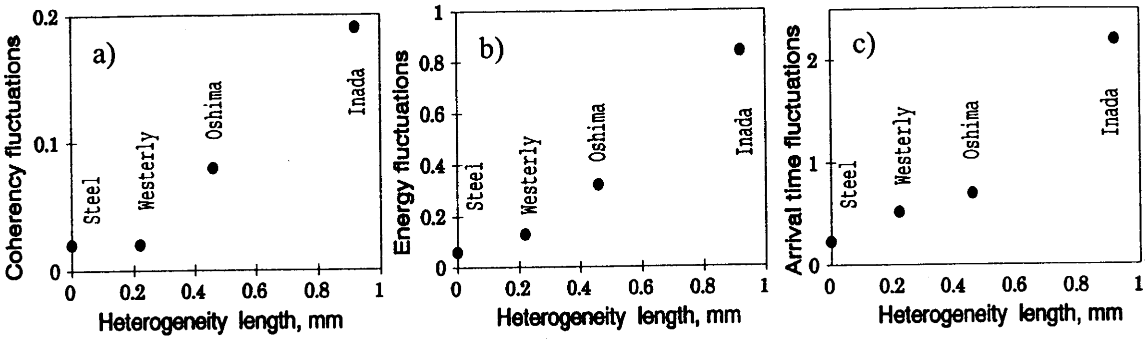

Figure 1. The inferred relationships of fluctuations and scale length of heterogeneity for the three parameters: a) correlation coefficient, r_i, b) energy and c) arrival times of the P-waves. The standard deviations of the respective fluctuations over the array are plotted for each medium.

Arrival Time fluctuations

We have adopted a method proposed by Takanami and Kitagawa (1991) for estimating the arrival time of the seismic wave. The technique involves fitting of autoregressive models to the time series using Akaike Information Criterion. In the autoregressive model, the present value of time series y_n is expressed as a linear combination of previous y_{n-i} (i=1,n) values as follows:

$$y_n = \sum_{i=1}^{m_j} a_{ji} y_{n-i} + v_{nj} \qquad (6)$$

where a_{ji} are the autoregressive coefficients of order m_j, and v_{nj} represents random variable with zero mean and variance, σ_j^2, in the j-th interval. The autoregressive coefficients of optimum order, m_j, are estimated by minimizing the variance between observed and fitted time series. Estimation of AR coefficients is effected by the order of AR model m_j; larger m_j provides narrower distribution of v_{nj} and the probability of realization becomes larger. However, increasing of m_j some times misleads the estimation of AR coefficients, because too much decrease of σ_j^2 gives misfit to the real v_{nj} distribution. A statistically sound trade off between increasing of m_j and decreasing of σ_j^2 is obtained by Akaike Information Criterion (AIC) as follows:

$$AIC_j = (N - m_1)(\log 2\pi + 1) + \sum_{j=1}^{k} N_j \log \sigma_j^2 + 2\sum_{j=1}^{k} (m_j + 1) \qquad (7)$$

where, k is the number of sub-intervals and N_j is length of the sub-interval. In order to study the relationship between the spatial fluctuation of arrival time, wave energy and the scale length of heterogeneity and the wave frequency, we computed the P-wave energy in each waveform using equation (4). The observed statistical distribution is obtained by computing frequency histograms of wave energy and arrival times for all observations in the array. The histograms are then fitted with theoretical probability distribution using Gaussian probability density function given by

$$F(x) = \frac{1}{2\pi\sigma_s} \exp\left(-\frac{(x-\mu)^2}{2{\sigma_s}^2} \right) \qquad (8)$$

where μ and σ_s are mean and standard deviation of energy or arrival times of the P-waves.

Discussion and Conclusions

The spatial coherency pattern and the energy attenuation patterns have been computed for each medium. These patterns exhibit distinct differences in media with different lengths of heterogeneity. The decay of coherency is steeper in media with large heterogeneity length than in media with small heterogeneity length. The energy decay pattern reveals that the energy of random wave increases with increasing scale length of heterogeneity. The relationships between the statistical quantities estimated from the waveform distortions and the characteristic length of heterogeneity of the medium are presented in Fig. 1. The variances of fluctuations of the correlation coefficient between coherent and the observed waves, fluctuations of wave energy and the fluctuations of arrival times increase with increasing scale length of heterogeneity. This depicts a positive correlation between the fluctuations of amplitudes, arrival times and the shape of the waveform and the characteristic dimension of heterogeneity of the medium.

References

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