Conventional N-, L- and R- tests

using no simulated catalogues

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International Symposium "Toward Constructing Earthquake Forecast Systems for Japan" ERI, 27 May, 2009

Assumptions

- Rate << 1
  at most 1 event occurs in a cell
- Independently occurs from other events
- Large number of events enough to use
  a Gaussian approximation
  (condition for the central limit theory)

Two kinds of distributions

- Distribution of the score for earthquakes conforming to a model under testing

- Distribution of the observable score if uncertainties in the hypocenter parameters taken into account

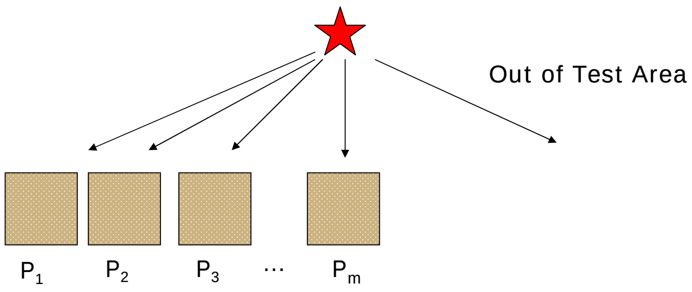
N-score

- *Reproductive property* of the Poisson process

$$\mathrm{E}[n] = \sum_{i=1}^{V_0} \lambda_i$$

$$\mathrm{Var}[n] = \sum_{i=1}^{V_0} \mathrm{Var}[n_i] = \sum_{i=1}^{V_0} \lambda_i$$

Uncertainties in hypocentral parameters



ΣP_i $\qquad$ $1 - \Sigma P_i$

Expected number : ΣP_i

Variance : $(\Sigma P_i)(1 - \Sigma P_i)$, binominal distribution

N- score

with uncertainties in the parameters

$$E[n_{obs}] = \sum_{j=1}^{N_0} \sum_{k=1}^{m_j} P_{j,k}$$

$$Var[n_{obs}] = \sum_{j=1}^{N_0} \left\{ \sum_{k=1}^{m_j} P_{j,k} \left(1. - \sum_{k=1}^{m_j} P_{j,k} \right) \right\}$$

L- score

expected from the model under testing

$l_i = Y_i \,\mathrm{Ln}\lambda_i + (1-Y_i)\,\mathrm{Ln}(1-\lambda_i)$

$\quad = Y_i \,\mathrm{Ln}\{\lambda_i / (1-\lambda_i)\} + \mathrm{Ln}(1-\lambda_i)$

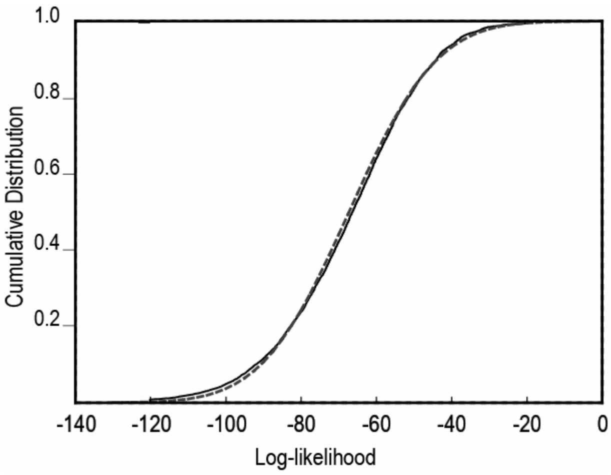
$$\mathrm{E}[l_i] = \lambda_i \,\mathrm{Ln}\{\lambda_i/(1-\lambda_i)\} + \mathrm{Ln}(1-\lambda_i)$$

$$\mathrm{Var}[l_i] = \mathrm{E}\left[\{\mathrm{Ln}\,\lambda_i/(1-\lambda_i)\}^2 (Y_i-\lambda_i)^2\right]$$

$$= \{\mathrm{Ln}\,\lambda_i/(1-\lambda_i)\}^2 \sum_{l=0}^{1} \left\{(y_l-\lambda_i)^2\,\mathrm{F}_i(y_l)\right\}$$

$$= \{\mathrm{Ln}\,\lambda_i/(1-\lambda_i)\}^2 \lambda_i$$

	Rate	N of Segments
1	0.010	100
2	0.008	125
3	0.005	400
4	0.004	500
5	0.002	1000
6	0.001	2000



L- score

with uncertainties in parameters

$$\Delta l_{j,k} = \mathrm{Ln}\{\lambda_{j,k}/(1-\lambda_{j,k})\}$$

Log-likelihood term contributed by k-th possibility of j-th event

$$\mathrm{E}(\Delta l_j) = \sum_{k=1}^{m_j} \mathrm{P}_{j,k}\Delta l_{j,k}$$

Log-likelihood perturbation by j-th event

$$\mathrm{E}(l) = \sum_{j=1}^{N_0}\sum_{k=1}^{m_j} \mathrm{P}_{j,k}\left(\mathrm{Ln}\frac{\lambda_{j,k}}{1-\lambda_{j,k}}\right) + \sum_{i=1}^{V_0}\{\mathrm{Ln}(1-\lambda_i)\}$$

$$\mathrm{Var}(l) = \sum_{j=1}^{N_0}\sum_{k=1}^{m_j} \mathrm{P}_{j,k}\{\Delta l_{j,k} - E(\Delta l_j)\}^2$$

R- score

expected from the model under testing

$$E[R^{12}] = \sum_{i=1}^{V_0} \left[\lambda^1{}_i \,\mathrm{Ln}\left(\lambda^1{}_i / \lambda^2{}_i\right) + (1 - \lambda^1{}_i)\,\mathrm{Ln}\left\{(1-\lambda^1{}_i)/(1-\lambda^2{}_i)\right\} \right]$$

$$\mathrm{Var}[R^{12}] = \sum_{i=1}^{V_0} \left[\mathrm{Ln}\left(\lambda^1{}_i / \lambda^2{}_i\right) - \mathrm{Ln}\left\{(1-\lambda^1{}_i)/(1-\lambda^2{}_i)\right\} \right]^2 \cdot \lambda^1{}_i \, .$$

R-test (Haz Map / EEPAS)